**Definition of a Dimension**

In a *d* – dimensional space, the volume of a ball of radius grows like , hence the following heuristic for dimension is obtained:

Some other properties that good definitions of dimensions should have are the following:

1. **Monotonicity**
2. **Stability**
3. **Countable Sets**
4. **Geometric Invariance**

**Minkowski Dimension**

This definition of dimension is a method of determining fractal dimension of a set *S* in .

Suppose the set *S* is placed on a grid of boxes with side length . Then, the Minkowski Dimension of *S* is:

where is the number of boxes needed to cover the set. When the limit doesn’t exist, the limit superior and limit inferior are used to define upper and lower Minkowski dimension.

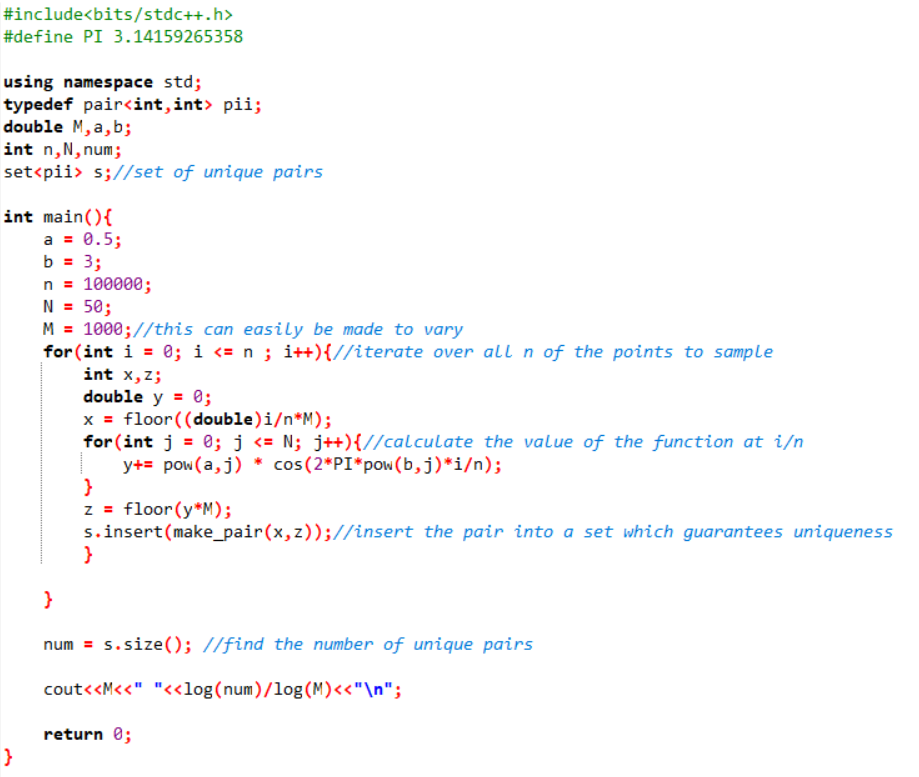
**The Algorithm**

Using Minkowski Dimension we tried to estimate the fractal dimension of the Weierstrass function, for particular values of *a* and *b*. We used

as an approximation of the function, and sampled a total of *n* points of the function on the interval .

The procedure was as follows:

1. Fix *n* , *N.*
2. Compute
3. Compute
4. Find

Example C++ Implementation:

Sources

“Minkowski–Bouligand Dimension.” Wikipedia, Wikimedia Foundation, 10 Apr. 2018, en.wikipedia.org/wiki/Minkowski%E2%80%93Bouligand\_dimension.

Pearse, Erin. “AN INTRODUCTION TO DIMENSION THEORY AND FRACTAL GEOMETRY: FRACTAL DIMENSIONS AND MEASURES.” AN INTRODUCTION TO DIMENSION THEORY AND FRACTAL GEOMETRY: FRACTAL DIMENSIONS AND MEASURES.

Tao, Terence. An Epsilon of Room. Pages from Year Three of a Mathematical Blog. American Mathematical Society, 2010.

**Results**

In order to estimate the Minkowski dimension, we graphed vs. (dimension). The results are shown in the two graphs below. In figure 1, the theoretical dimension is 1.37 and in figure 2 it is 1.50. The approximations were close but did not seem to converge to the theoretical value. (For all calculations N = 50).

Another approach we tried was varying the number of sampled points *n*, while fixing a grid size based on *n*.

|  |  |  |
| --- | --- | --- |
|  | Dimension |  |
| *n* | *a = 0.5, b = 3*  *(theoretical value 1.37)* |  |
| 100000 | 1.7537 |  |
| 1000000 | 1.6924 |  |
| 10000000 | 1.6455 |  |

This approach tends to overshoot dimension so it’s possible that larger values of *n*, will yield better estimations. However, testing larger *n* was not possible due to the time complexity of the algorithm.

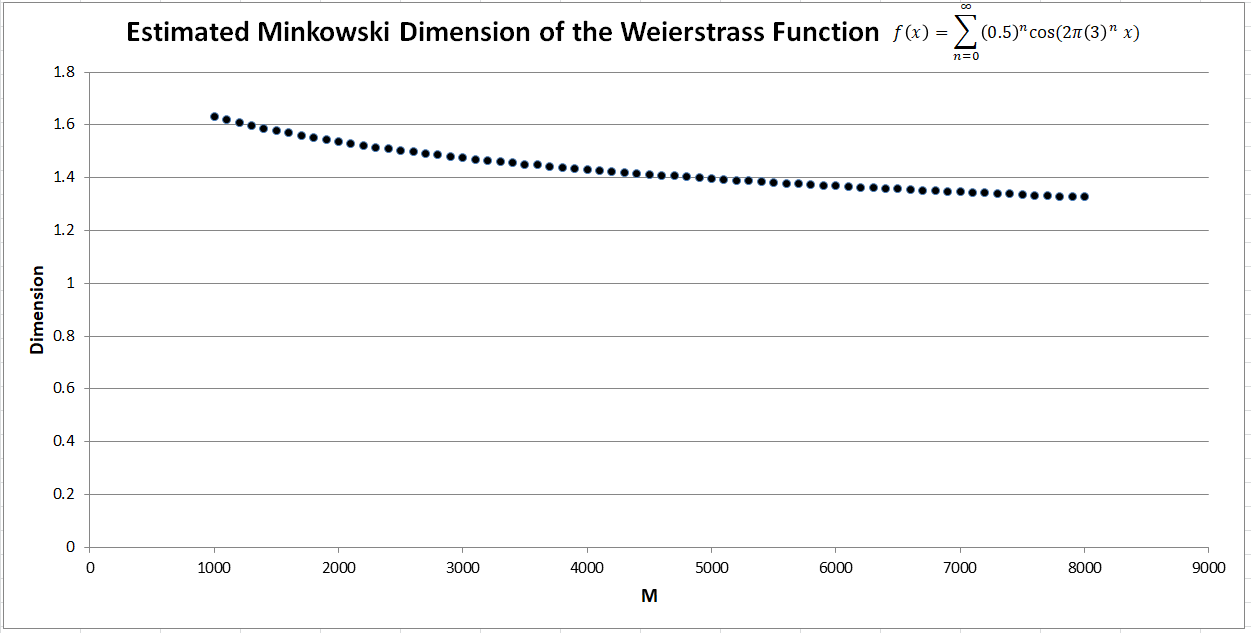
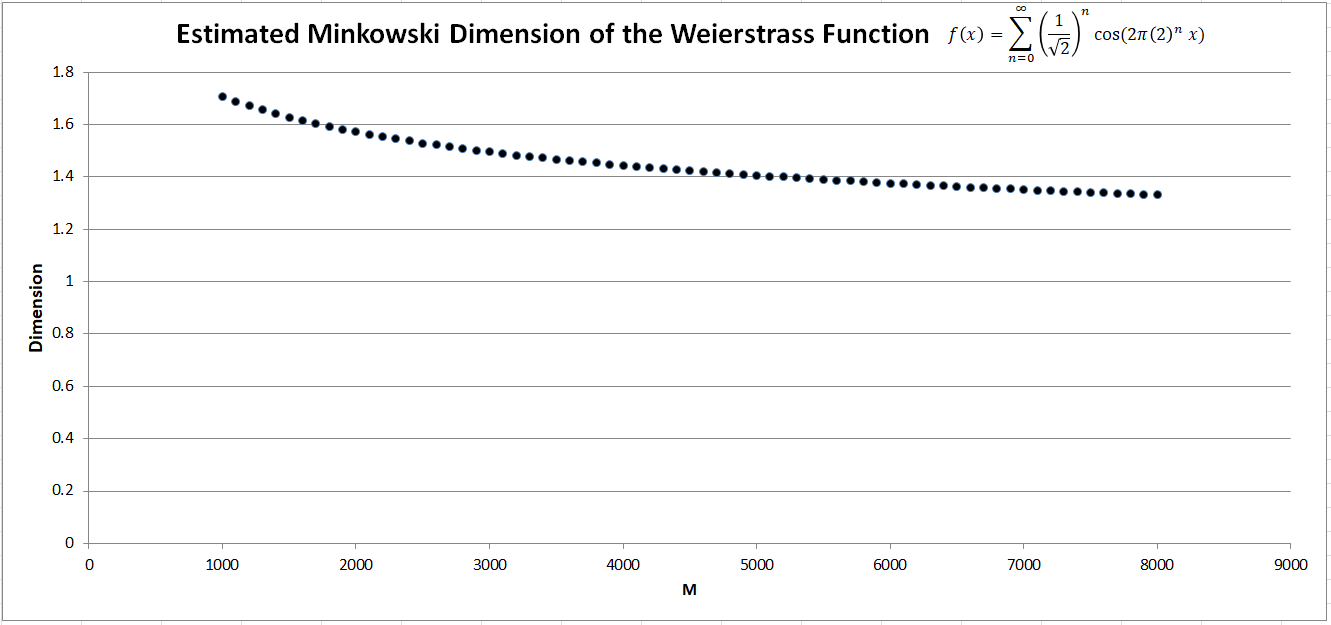


Figure 2

Figure 1